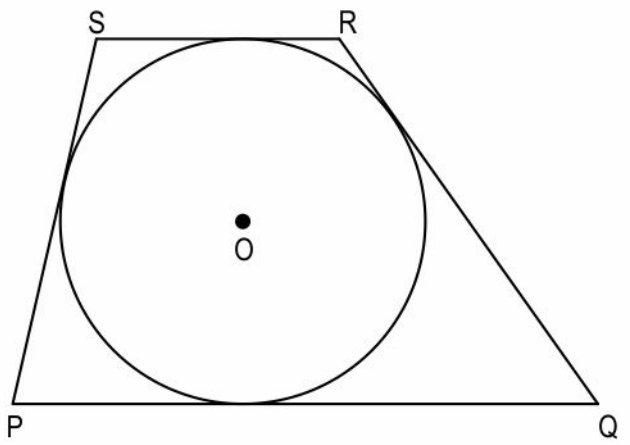
Circles

Free Response Questions

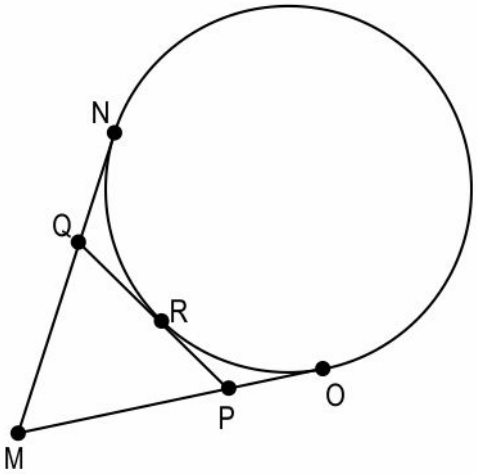
Q: 1 **Shown below is a circle O inscribed in a trapezium such that PQ || RS. The radius of the circle is 6 cm.** **[1]**



(Note: The figure is not to scale.)

What is the height of the trapezium? Show your work and give valid reasons.

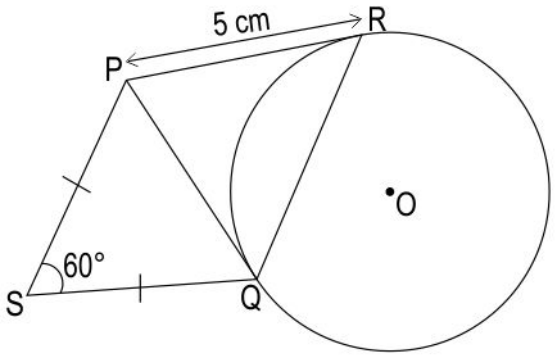
Q: 2 **Shown below is a circle with 3 tangents at points N, O and R. QN = PO = *x* units and ▲QMP is an equilateral triangle.** **[1]**



(Note: The figure is not to scale.)

Express the length MN in terms of *x*. Show your work.

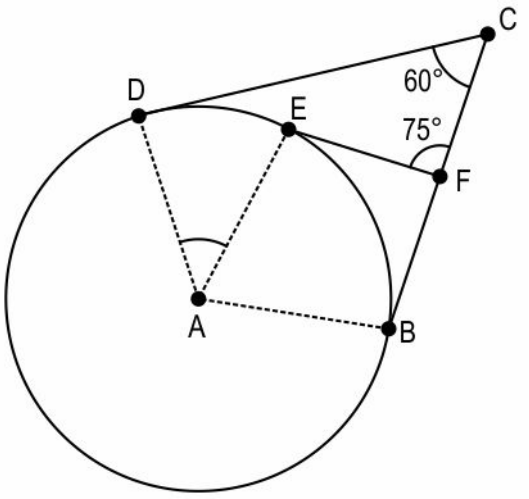
Q: 3 **In the figure below, PQ and PR are tangents to the circle.** **[1]**



(Note: The figure is not to scale.)

Find the perimeter of ▲SQP. Show your work.

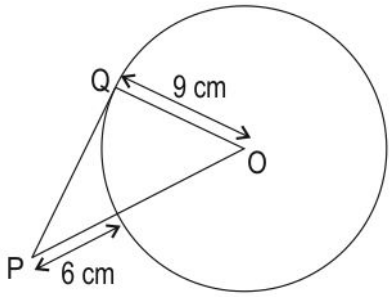
Q: 4 **Shown below is a circle with centre A. CD, CB and FE are tangents to the circle. ∠DCF = 60° and ∠EFC = 75°.** **[2]**



(Note: The figure is not to scale.)

Find the measure of ∠DAE. Show your work and give valid reasons.

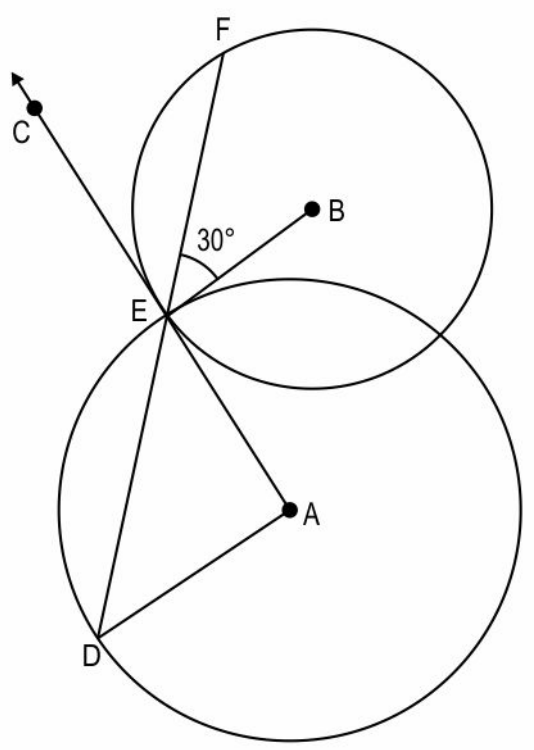
Q: 5 **Shown below is a circle with centre O. PQ is tangent to the circle at Q.** **[2]**



(Note: The figure is not to scale.)

What is the length of tangent PQ? Show your work and give valid reasons.

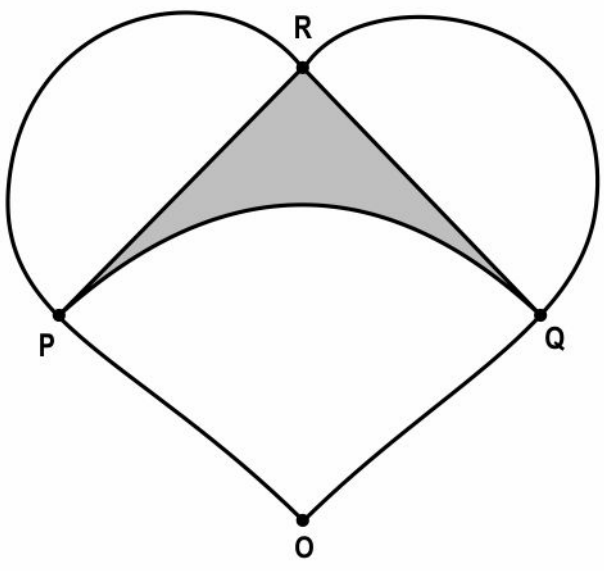
Q: 6 **Shown below are two circles with centres A and B. The circle with centre A has a radius of 5 cm. AC is a tangent to circle with centre B. FED is a straight line.** **[3]**



(Note: The figure is not to scale.)

What is the length of chord DE? Show your work and give valid reasons.

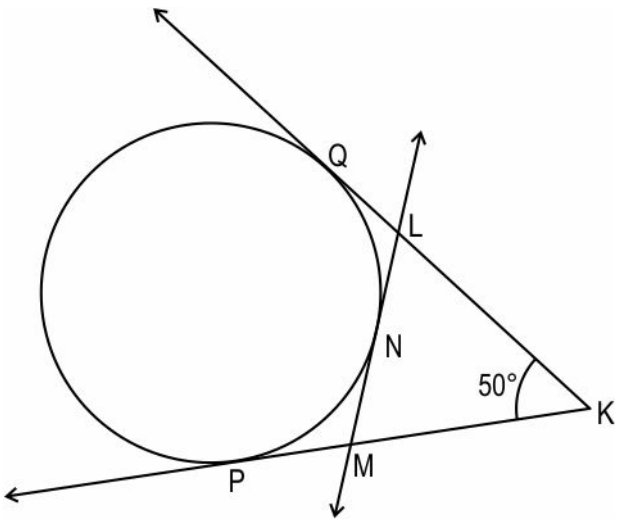
Q: 7 **In the figure below, PR and QR are tangents to a circle with centre O, at points P and Q. Two semicircles are drawn on tangents PR and QR. Each semicircle has an area of π square units. Quadrant POQ has an area of 2π square units.** **[3]**



(Note: The figure is not to scale.)

Find the area of the shaded region. Show your work and give valid reasons.

Q: 8 **Shown below is a circle with 3 tangents KQ, KP and LM.** **[3]**



(Note: The figure is not to scale.)

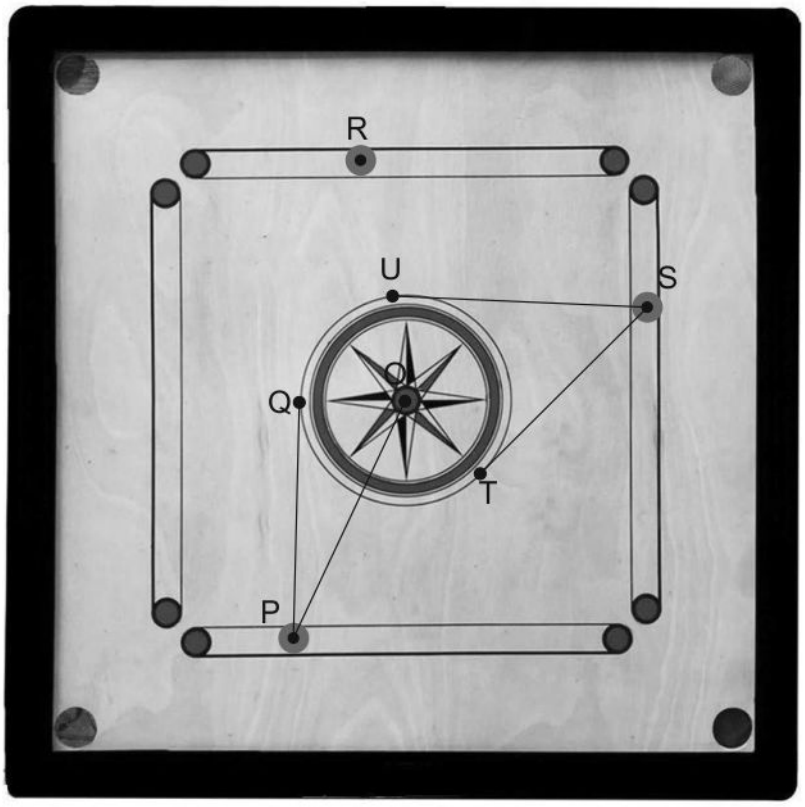
If PM:KL = 1:2, find the measure of ∠LMK. Show your steps.

Case Study

Answer the questions based on the given information.

The game of carrom is played on a square board with a pocket in each corner. They are played using small disks of wood or plastic known as carrom coins. A larger and heavier disk called a striker is used to push these coins across the board to the pockets. The positions of striker and a few coins at different instances in a game are shown in the figure:

P, R and S denote different positions of the striker.
Q, T and U denote different positions of the coins.
O is the centre of the carrom board.



The points P, Q and R do not lie on a straight line. The line PQ touches the circular part of the carrom board only at Q.

Q: 9 **The striker at P is 18 cm from the centre and ∠OPQ = 30°.** **[2]**

Find the distance between the striker at P and the coin at Q. Show your work and give valid reasons.

Q: 10 **Can QR be a tangent to the circle at Q? Give a valid reason.** **[1]**

Q: 11 **SU and ST are tangents to the circle at U and T respectively and ∠UQT = 55°.** **[2]**

Find ∠UST. Show your work and give valid reasons.

Q.No	What to look for	Marks
1	**Writes that the radius is perpendicular to the tangent at its point of contact, hence the diameter is the shortest distance between PQ and RS.**	**0.5**
	Finds the height of the trapezium as 2 × 6 = 12 cm.	**0.5**
2	**Identifies PR = PO = x, QR = QN = x and finds the length of PQ as 2 x units.**	**0.5**
	Identifies QM = PM = PQ = 2 x units and finds the length of MN as 2 x + x = 3 x units.	**0.5**
3	**Finds the length of PQ = PR = 5 cm.**	**0.5**
	Concludes that $\triangle$SQP is an equilateral triangle and finds its perimeter as 3 × 5 = 15 cm.	**0.5**
4	**Writes that angles on a straight line are supplementary and finds the measure of $\angle$EFB as 180° - 75° = 105°.**	**0.5**
	Writes that a tangent to a circle is perpendicular to the radius through the point of contact and hence, $\angle$AEF = $\angle$ABF = $\angle$ADC = 90°.	**0.5**
	Writes that sum of angles in quadrilateral ABFE is 360° and finds the measure of $\angle$EAB as 360° - 285° = 75°.	**0.5**
	Writes that sum of angles in quadrilateral ABCD is 360° and finds the measure of $\angle$DAE as 360° - 315° = 45°.	**0.5**
5	**Finds the length of PO as 6 + 9 = 15 cm.**	**0.5**
	Writes that a tangent to a circle is perpendicular to the radius through the point of contact and hence $\angle$PQO = 90°.	**1**
	Uses pythagoras theorem in $\triangle$PQO to find the length of tangent PQ as $\sqrt{(15^2 - 9^2)}$ = 12 cm.	**0.5**
6	**Writes that the radius is perpendicular to the tangent at the point of contact and finds $\angle$FEC as 90° - 30° = 60°.**	**1**

Q.No	What to look for	Marks
	Writes that alternate angles are equal and hence ∠AED = ∠FEC = 60°.	**0.5**
	Identifies that AD = AE and hence ∠AED = ∠ADE = 60°.	**0.5**
	Writes that sum of angles in a triangle is 180°, hence ∠DAE = 60°.	**0.5**
	Concludes from the above steps that ▲ADE is equilateral and hence the length of chord DE is 5 cm.	**0.5**
7	**Assumes the radius of the semicircles as *r* units and finds the diameter of the semicircles as:** $\frac{\pi r_2}{2} = \pi$ **=> 2 *r* = 2√2 units**	**0.5**
	Assumes the radius of the quadrant as *q* units and finds the radius of the quadrant as: $\frac{\pi q_2}{4} = 2\pi$ **=> *q* = 2√2 units**	**0.5**
	Writes that quadrilateral POQR is a square as all four sides measure 2√2 units and all vertex angles measure 90°.	**1**
	Finds the area of the square as 2√2 × 2√2 = 8 square units.	**0.5**
	Finds the area of the shaded region as (8 - 2π) square units.	**0.5**
8	**Assumes the lengths of PM as *x* units and KL as 2 *x* units. Writes the length of MN as *x* units.**	**0.5**
	Assumes the length of QL = LN as *y* units and writes the length of QK as (2 *x* + *y*) units.	**1**
	Writes the length of LM as MN + LN = (*x* + *y*) units.	**0.5**
	Writes the length of MK as (2 *x* + *y*) - *x* = (*x* + *y*) units.	**0.5**

Q.No	What to look for	Marks
	Writes that since LM = MK, the measure of ∠LMK = 180 - (50 + 50) = 80°.	**0.5**
9	**Finds that OQ is the radius of the circle and hence OQ is perpendicular to PQ.**	**0.5**
	Writes that in ▲OPQ: **PQ = OPcos 30° = 18 × $\frac{\sqrt{3}}{2}$**	**1**
	Simplifies the above expression and finds the distance between the striker at P and the coin at Q as 9√3 cm.	**0.5**
10	**Writes that only one tangent can be drawn from a point on the circle and since PQ and QR are two different straight lines and PQ is a tangent, QR cannot be a tangent to the circle at Q.**	**1**
11	**Finds ∠UOT as 110° and states that angles subtended by an arc at the centre of the circle is double the angle subtended by that arc anywhere on the circle.**	**1**
	Finds that ∠UST = 180° - 110° = 70° as OU is perpendicular to SU, OT is perpendicular to ST and angle-sum property of a quadrilateral.	**1**